SEISMIC RESERVOIR CHARACTERIZATION IN OFFSHORE NILE DELTA. PART I: COMPARING DIFFERENT METHODS TO DERIVE A RELIABLE ROCK-PHYSICS MODEL

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Introduction. Seismic-reflection data are used in reservoir characterization not only for obtaining a geometric description of the main subsurface structures but also for estimating properties like lithologies and fluid contents of the target levels of interest. To this end, a rock-physics model (RPM) is incorporated into a seismic inversion scheme, such as amplitude versus angle (AVA) inversion (Grana and Della Rossa, 2010) or full-waveform inversion (Bacharach, 2006), to directly derive petrophysical rock properties from pre-stack seismic data. The outcomes of petrophysical-seismic inversion provide reservoir property maps to reservoir engineers for field appraisal, selection of optimal well location, and production enhancement (Bosh *et al.,* 2010). A rock-physics model is a generic transformation (f_{RPM}):

$$\left[Vp, Vs, Density\right] = f_{RPM}(\phi, Sw, Sh, z) \quad (1)$$

The RPM relates the rock properties (which typically are porosity - ϕ -, water saturation - *Sw* - , shale content - *Sh* -) and depth (*z*) to the pressure conditions, to elastic attributes (such as P-wave and S-wave velocities - *Vp*, *Vs* - and density). A rock-physics model can be based on theoretical equations (Avseth *et al.,* 2005), or on empirical set of equations derived from available information (e.g. well-log or core measurements) for the specific case of interest

(Mazzotti and Zamboni, 2003). In the last case, we considered either a linear or a non-linear model (Eberhart-Phillips *et al.,* 1989). In the non-linear approach, many methods can be used to derive such rock-physics model. Neural networks (Saggaf *et al.,* 2003) and stochastic optimizations (Aleardi, 2015) have received great attention. Anyway, independently from the method used, there is no doubt that the quality and the reliability of available well-log data and/or core measurements play an essential role in defining a solid RPM.

The aim of this work is to derive a reliable RPM used in conjunction with an AVA inversion for the characterization of a clastic reservoir located in offshore Nile delta. We have employed both theoretical and empirical approaches to derive the RPM. For what concerns the empirical approaches we used both a linear and two non-linear methods to define different rock-physics models. We obtained the linear model by applying a multilinear stepwise regression, whereas neural networks and genetic algorithms are used to derive non-linear transformations from petrophysical to elastic properties. The main difference among neural networks and genetic algorithms is that the former is a gradient-based method while the latter is a global, stochastic, optimization method.

We start by introducing the different methods used to derive the theoretical and the empirical rock-physics models. Then, the detailed analysis of RPMs resulting from theoretical and empirical approaches let us to outline the benefits and the limits of each method. Moreover, in the empirical approaches we focus our attention on discussing the differences between linear and non-linear methods for the specific case under examination and on analyzing the drawbacks that characterize the neural network technique. The simplicity and the reliability of the empirical rock-physics model derived by applying multilinear stepwise regression and the optimal prediction capability of the theoretical rock-physics model enable us to consider these two RPMs in the petrophysical AVA inversion that is discussed in the companion paper titled "*Seismic reservoir characterization in offshore Nile Delta. Part II: Probabilistic petrophysical-seismic inversion*".

A brief introduction to the methods used for deriving the rock-physics models. In this chapter, we briefly describe the empirical and the theoretical methods used to derive the rock-physics models. We start with the multilinear stepwise regression followed by the neural network approach, and by the optimization of the genetic algorithm used in the empirical approach. Thereafter, we will introduce the theoretical approach based on rock-physics models.

Multilinear stepwise regression (SR). Stepwise regression is a semi-automated process of deriving a linear equation by successively adding or removing variables in the regression procedure based solely on the *t*-statistics of their estimated coefficients (Draper and Smith, 1985). Three main approaches can be used in this regression method: forward selection, backward elimination and bidirectional elimination. The first approach starts with no variable in the model and proceeds forward (adding one variable at a time). The second approach starts with all potential variables in the model and proceeds backward (removing one variable at a time). In this study, we applied the third method, which is a combination of the approaches described above and is essentially a forward selection procedure but allows the elimination of a selected variable at each optimisation stage.

Neural Network optimization (NN). A neural network is a mathematical algorithm inspired by an animal's central nervous system and trained to solve problems that would normally require human intervention (Haykin, 1999). In particular, a supervised neural network corresponds to a problem in which a set of input data and their corresponding outputs are available; in this way, the network can attempt to infer a relation between the input and the output. In this work, we apply a multilayer feed-forward neural network in which the petrophysical rock properties define the input, whereas the elastic attributes are the output of the net. The architecture of the network we used consists of one input layer, one output layer, and one hidden layer. In this work, the hidden layer consists of 25 neurons, whereas the input has as many nodes as the petrophysical properties of interest. The nodes in each layer are characterized by a sigmoid transfer function. The weights associated with each node are computed such that the value at the output layer is equal to the training value in the least-squares sense. The NN optimization is essentially a steepest descent algorithm iteratively adjusting initial random weights using a technique called back-propagation (Haykin, 1999). For a review of neural networks and their geophysical applications, see van der Baan and Jutten (2000).

Genetic algorithms (GA) optimization. Genetic algorithms are a stochastic optimization method based on the mechanics of natural selection and evolution according to the Darwinian principle of "survival of the fittest" (Holland, 1975). In a GA optimization procedure a population of randomly generated individuals, which represent candidate solutions to an optimization problem, is evolved toward better solutions applying three main genetic operators, which are selection, cross-over and mutation. For more details see Mitchell (1996). In this work, we apply a GA optimization in which a population of 100 individuals evolves into 50 iterations. In this GA optimization, the equations describing the RPM are:

$$EP = k + \sum_{n=1}^{N} a_n PP_n^{\,b_n} \tag{2}$$

where: *EP* represents a generic elastic property (e.g. P-wave velocity, density…), *PP* is a generic petrophysical property (e.g. porosity, water saturation…) and *N* is the number of petrophysical properties considered in the regression process. The weight of each input variable is given by the coefficient *a,* the exponent *b* is used to reproduce the effects of variations in the petrophysical properties on the elastic property under consideration**,** whereas *k* is the intercept of the final equation. The coefficients (k; a_1, a_2,..., a_N; b_1, b_2,..., b_N) are contained in each individual that is evolved during the GA optimization in which the L_2 norm between observed and predicted elastic properties defines the error function to be minimized. Note that Eq. 2 represents a generalization of classical depth trends (Banchs *et al.,* 2001).

Theoretical rock-physics model (TRPM). With theoretical rock-physics model we refer to one or more theoretical equations that establish a relationship between elastic attributes and petrophysical rock properties. To this end, several models exist (e.g. granular media models and inclusion models). The reader can find an extensive discussion of TRPM in Avseth *et al*. (2005) and Mavko *et al*. (2009). In this work, following Avseth *et al*. (2005), we use the Hertz-Mindlin theory to define the shale and sand dry elastic properties at critical porosity and hydrostatic pressure. To simulate the compaction effect we used the Hashin-Strikmann lower bound, whereas the Gassmann equation defines the saturated elastic properties. Taking into account the depth interval considered in this study, characterized by a mechanical compaction regime, we assume a shale totally formed by smectite mineral and a not-cemented sand totally formed by quartz grains. The shale and sand critical porosities are fixed to 70% and 40%, respectively.

Results. In this chapter, we analyze and comment the RPMs resulting from theoretical (TRPM) approach and from linear (SR) and non-linear (NN and GA) empirical approaches. The well-log data used to estimate the RPMs pertain to four exploration wells drilled through the reservoir zone (sand) and the encasing non-reservoir rocks (shale). The petrophysical (*Sh, Sw,* ϕ) and elastic (*Vp, Vs, density*) properties, that we consider, are all derived from appropriate formation evaluation analysis of actual well-log measurements and have been subjected to an accurate outlier removing procedure.

We know that non-linear relations often relate petrophysical properties and elastic characteristics. For example, non-linear relations link the shale content to *Vp* and *Vs* and the water saturation to *Vp* if considered in their full range from 0% to 100% (see Avseth *et al.,* 2005 for more details). However, in order to investigate the capability of a linear method in deriving a reliable rock-physics model, in this specific case, we consider the entire shale content and water saturation ranges (from 0% to 100%). Differently, the depth interval is limited to the target sands and the encasing shales and ranges from 2400 to 3000 m, approximately.

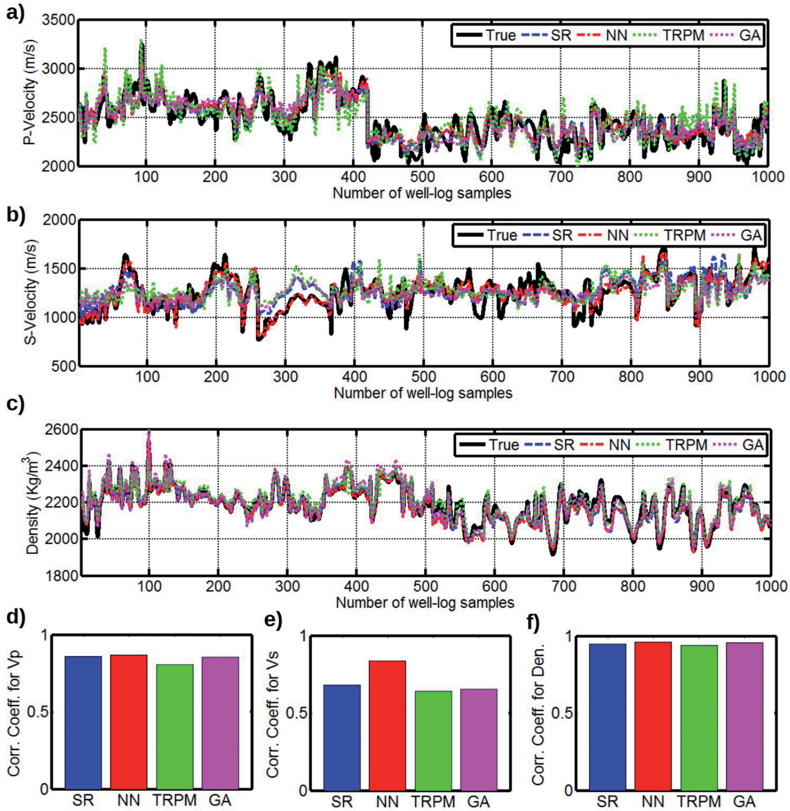

Fig. 1 – Comparison between the true and the predicted elastic properties. Vp, Vs and density are represented in (a), (b) and (c), respectively. The correlation coefficients computed for each approach are shown in (d), (e) and (f) for Vp, Vs and density, respectively.

A first assessment of the estimated RPMs can be done by comparing the true elastic properties, derived from well-log recordings, with those predicted by including the actual petrophysical data in the RPMs. The Fig. 1 shows this comparison together with the resulting correlation coefficients. First, we note that independently from the method applied (SR, NN, GA or TRPM) the quality of the prediction decreases passing from density (Fig. 1c) to *Vp* (Fig. 1a) and to *Vs* (Fig. 1b). Indeed, as it is well known, the relation linking the density to the petrophysical properties is simpler than the relations linking these properties to *Vp* and *Vs* (Aveseth *et al*. 2005). The higher correlation coefficient obtained for *Vp* (Fig. 1d) with respect to *Vs* (Fig. 1e) can be explained with the lower performance of the logging tools in measuring the S-velocity, and then the lower reliability of the *Vs* measurements with respect to the *Vp* and the density ones. For the density, the four methods return very similar results. Differently, for *Vp* and, particularly, for *Vs* the NN method yields final estimates with the highest correlation coefficients, whereas slightly lower correlation coefficient are obtained by the SR, GA and the TRPM methods. The slightly higher correlation coefficients obtained by the empirical methods with respect to the TRPM can be easily explained taking into account that the empirical approaches are data-driven procedures, and thus derive the final f_{RPM} on the basis of the actual petrophysical and elastic properties. Conversely, the TRPM is based on theoretical equations with general validity. Among the empirical approaches, NN produces a slightly better match than SR and GA, whereas the non-linear GA method and the multilinear SR algorithm return very similar predictions. The main advantage of the SR and GA methods over the NN approach is that they directly provide equations (relating the petrophysical to the elastic properties) with an easily interpretable rock-physical meaning. Conversely, the NN result is a sort of "*black box*".

In this work, the SR approach returns the following equations:

$$Vp\left[m/s\right] = 1732.3 + 0.542\,z + 1.647\,Sw - 28.742\phi - 9.056\,Sh \tag{3a}$$

$$Vs\left[m/s\right] = 1198.2 + 0.514\,z - 2.951\,Sw - 36.072\phi - 11.241\,Sh \tag{3b}$$

$$Density\left[Kg/m^3\right] = 2362.4 + 0.073\,z + 1.257\,Sw - 17.351\,\phi - 3.746\,Sh \tag{3c}$$

whereas with GA we obtain:

$$Vp\left[m/s\right] = 1685.5 + 0.623\,z^{0.94} + 1.831\,Sw^{1.16} - 30.142\phi^{0.87} - 8.547\,Sh^{1.04} \tag{4a}$$

$$Vs\left[m/s\right] = 1231.4 + 0.473\,z^{0.99} - 2.479\,Sw^{1.05} - 34.492\phi^{1.13} - 13.276\,Sh^{0.86} \tag{4b}$$

$$Density\left[Kg/m^3\right] = 2482.1 + 0.105\,z^{0.83} + 1.22\,Sw^{0.96} - 17.936\,\phi^{1.06} - 3.364\,Sh^{1.17} \tag{4c}$$

where: the depth (z) is expressed in meters and Sw, ϕ and Sh in percentage. We note that the intercepts and the coefficients in Eqs. 3 and 4 are very similar. In addition, the exponents in Eq. 4 are very close to one. These characteristics enable us to conclude that, in the specific case under examination, the relations linking the petrophysical to the elastic properties are close to be linear. This is confirmed by the very similar predictions returned by GA and SR (as previously evidenced in commenting Fig. 1). The possibility of describing the RPM by means of the linear equations will simplifies the uncertainty propagation in the probabilistic petrophysical-seismic inversion discussed in the aforementioned companion paper. From Eqs. 3 and 4 we note that, as expected, Vp, Vs and density increase as the depth increases (the z parameter has always a positive coefficient), and decreases as the porosity increases (the ϕ parameter has always a negative coefficient). Hydrocarbons are usually characterized by low bulk modulus and density and this fact explains the Vp and density increases as the water saturation increases (in Eqs. 3a, 3c and 4a, 4c, the Sw parameter has a positive coefficient). Conversely, the shear modulus is not affected by the saturating fluid and this fact, together with the density decrease produced by the increase of hydrocarbon saturation, explains the Vs increase as the water saturation decreases (in Eqs. 3b and 4b, the Sw parameter has a negative coefficient). The negative coefficients associated to the shale content can be related to the specific depth interval considered in this work (2400-3000 m), characterized by a mechanical compaction regime. As discussed in Avseth *et al*. (2005) in this depth interval the P-wave and S-wave velocities and the density of shales are usually lower than those of sands. Focusing our attention on the coefficients associated to the petrophysical variables we conclude that the porosity plays the major role in determining the elastic properties, followed by the shale content and the water saturation. Therefore, in the following seismic-petrophysical inversion, we expect that the porosity and, secondarily, the shale content will be the best determined parameters, whereas water saturation will be poorly resolvable (see the companion paper).

In Fig. 2 we have represented the so called rock-physics template (RPT) (Avseth *et al.,* 2005). The RPT is a cross-plot that shows the influence of each petrophysical property on the elastic attributes (in this work we consider the P-impedance (Ip) and S-impedance (Is)). Fig. 2a shows the RPT derived from the actual well-log data and the associated petrophysical properties. The well-known hydrocarbon trend is also represented. As expected, we observe a decrease of Ip and an increase of Is as the water saturation decreases and a decrease of both Ip and Is with the increasing of porosity and with the increasing of shale content. These general trends are well matched by the RPTs derived from the empirical (Figs. 2b, 2c and 2e) and the theoretical (Fig. 2c) RPMs. As previously discussed, the RPTs estimated by GA and SR are very similar, and even in this case the NN method yields a better match with the actual RPT

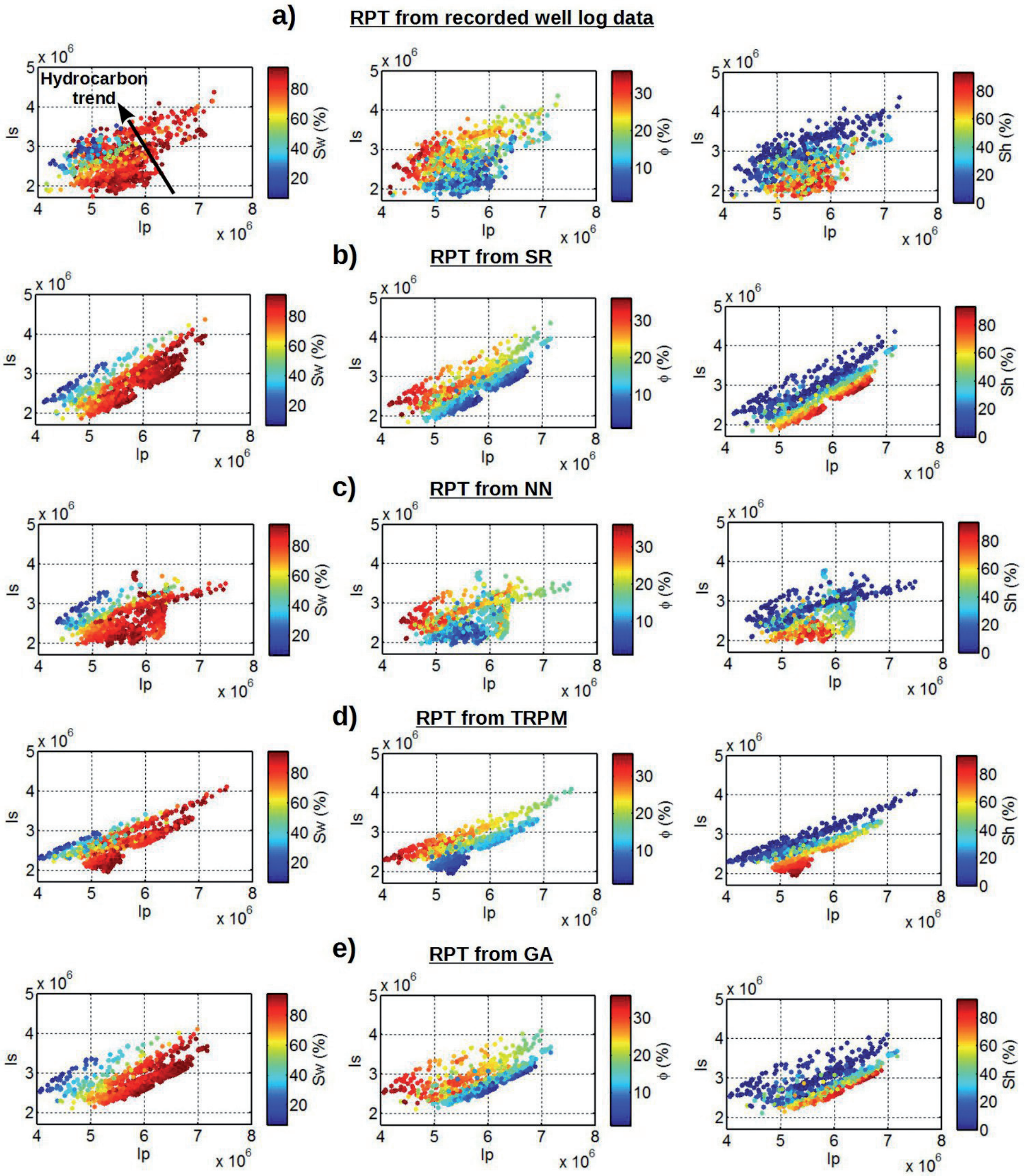

Fig. 2 – Rock-physic templates showing the influence of each petrophysical parameter on the P-impedance (Ip) and S-impedance (Is). Water saturation (Sw), porosity (ϕ) and shale content (Sh) are represented from left to right. Part (a) refers to the actual well-log data, whereas parts (b), (c), (d) and (e) refer to the elastic properties predicted by the SR, NN, TRPM and GA methods, respectively. In part a) the hydrocarbon trend is indicated by the black arrow.

computed from the actual well-log data. A slightly lower match characterizes the SR and the GA estimates and, particularly, the TRPM results.

The actual benefits and drawbacks of each method can be seen in Fig. 3. In Figs. 3a, 3b, 3c and 3d we show a graphical representation of the rock-physics models previously derived. SR, NN, TRPM and GA outcomes are depicted from Figs. 3a to 3d, respectively. For the lack of space, we restrict our attention to the *Vs* parameter and to the porosity-shaliness plane only. The great similarity between the SR and the GA results can be observed by comparing Figs.

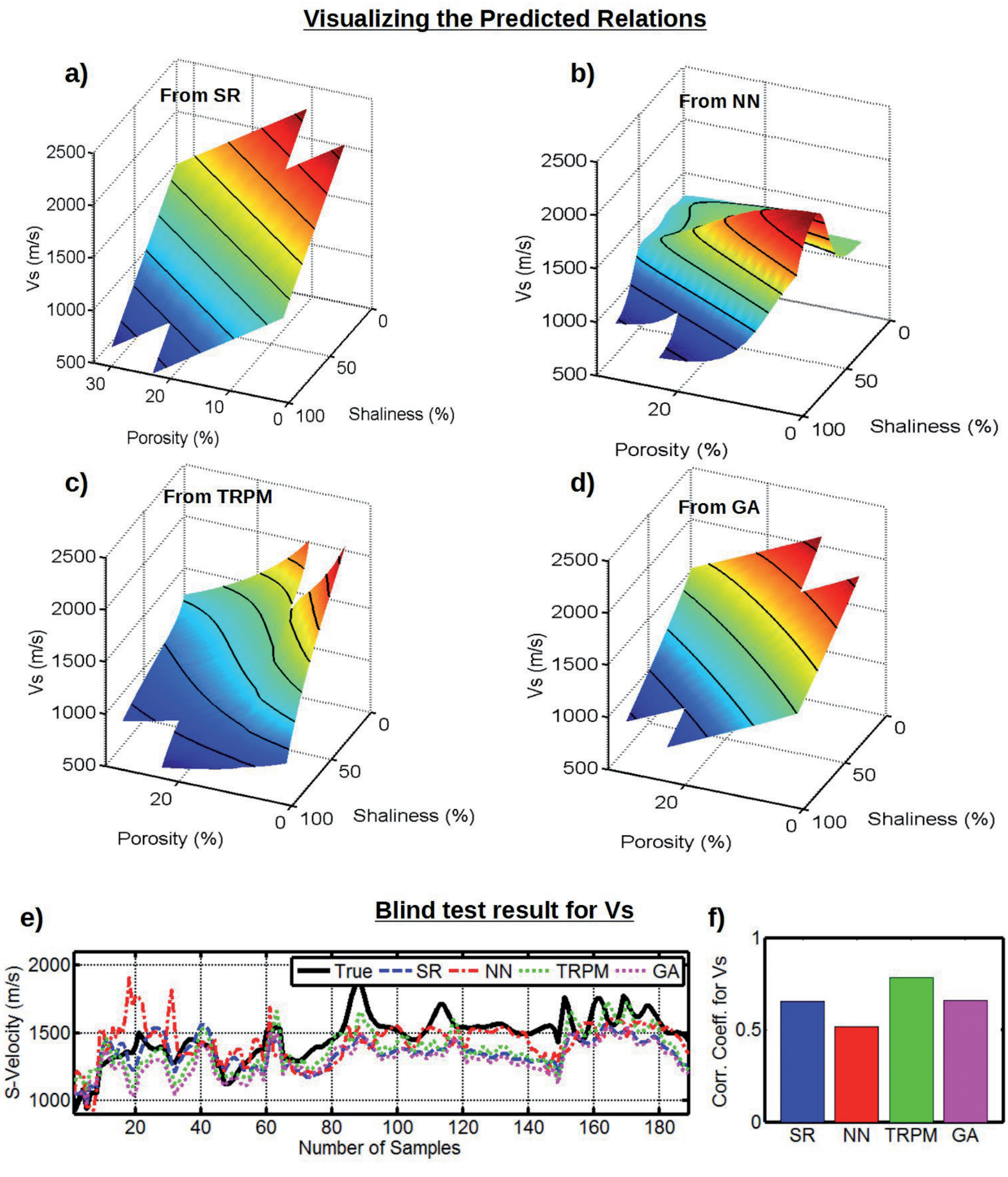

Fig. 3 – Graphical representations of the rock-physics models derived by step-wise regression (SR), Neural Network (NN), theoretical rock physics model (TRPM) and genetic algorithms (GA) are, respectively, shown in (a), (b), (c) and (d). These surfaces represent the Vs variations as a function of the shale content and the porosity, keeping fixed the depth and the water saturation to 2700 m and to 50%, respectively. (e) and (f) Results of the blind test and the corresponding correlation coefficients. See the text for additional comments.

3a and 3d. In addition, we note a fair similarity between the rock-physics models derived by the empirical SR and GA methods and the one resulting from the TRPM. All these three RPMs predict similar *Vs* increases with the decreasing of shale content and similar *Vs* increases as the porosity decreases. Conversely, the rock-physics model obtained by the NN method is substantially different from the other ones: it shows an un-physical *Vs* decrease for a shale content less than 40-50%, approximately. We interpret this fact as an overfitting problem that usually affects the NN method (see van der Bann and Jutten, 2000). In particular, in the context of a NN optimization, an excessively complex network generates overfitting. Generally, overfitting occurs when a model describes random error or noise instead of the underlying relationship. A model that has been overfitted will generally have poor predictive performance. In the specific case, the overfitting associated with the NN method is visible by comparing Figs. 2a and 2c. In these figures, we note that the RPT derived from the NN approach tends to reproduce the scatter visible in the RPT derived from the recorded logs. Such scattered trend is clearly related to residual noise contamination in the well-log data or inaccurate measurements and it has no physical meaning. As a clarifying demonstration of the overfitting problem, we perform a blind test in which the petrophysical relations expressed by the four RPMs are used to predict the elastic properties in a nearby well that was not used in the estimation process of the RPMs (Fig. 3e). This well was drilled in the same target area and through geological formations with similar characteristics. This test is also aimed at quantifying the prediction capability of each rock-physics model. For the lack of space, we show the results of the blind test obtained for the S-velocity only that is the more difficult parameter to predict as demonstrated in Fig. 1. In Fig. 3e we note that, thanks to its general validity, the TRPM approach gives the best fit with the actual data. Conversely, the empirical, data-driven, approaches show lower correlation coefficients than TRPM. In particular, the NN approach is characterized by the lowest correlation coefficient (Fig. 3f), thus confirming that the overfitting problem is often associated with a sub-optimal prediction capability. Moreover, the blind test allows us to discuss a fundamental difference between the empirical and the theoretical approaches, that explains the lower correlation coefficients that characterize all the empirical RPMs with respect to the theoretical one (Fig. 3f). Even if the input set of elastic and petrophysical properties, used in defining the rock-physics model, belong to wells drilled through geological formations with similar characteristics, the empirical, data-driven, approaches return slightly different models depending on the set of input data considered in the prediction procedure. This fact can be ascribed to errors and uncertainties that affects the measured elastic properties and to errors and approximations made in the formation evaluation analysis to derive the petrophysical properties. Differently, the TRPM result, being based on theoretical equations, is totally independent from errors and uncertainties in well-log measurements.

Conclusions. We have analyzed the rock-physics models (RPMs) obtained by applying both theoretical and empirical approaches. The fair match between the measured and predicted elastic properties and between the actual and the predicted RPTs demonstrates the potential of all the considered methods to yield final equations that are capable of estimating the elastic properties from a set of input petrophysical properties. A very high correlation coefficients characterize the density estimates, whereas lower correlation coefficients characterize the predicted seismic velocities. This fact evidences that the relation that links density to the petrophysical parameters is simpler than the relations existing between the petrophysical parameters and *Vp* and *Vs*. In addition, the lower correlation coefficient observed for the *Vs* estimates might be due to the lower performance of the logging tools in measuring the S-velocity.

We have shown that the non-linear GA and the linear SR methods return very similar equations, demonstrating that the relations linking the input petrophysical properties to the elastic attributes are, in this specific case, close to be linear. This fact makes the application of an empirical non-linear method useless for the case under examination. However, in more complex geological settings the linear approach may not be enough to ensure a good match between measured and predicted properties. In these cases non-linear methods should be applied. Among the non-linear methods we tested, an important limitation of the NN over the GA method is the overfitting problem. Another drawback of the NN method, not analyzed here, is related to its local nature. In a NN optimization, the weights associated with each neuron are usually randomly initialized and are subsequently adjusted using a gradient-based strategy. This demonstrates the importance of a good initial model to prevent convergence towards a local minimum in the case of a complex multiminima error function. Conversely, GA method (or other global search algorithms) circumvents this drawback by performing a wide and efficient exploration of the entire model space in a single inversion. Moreover, the main advantage of GA and SR over NN is that they directly provide simple equations characterized by easily interpretable rock-physical meanings.

The blind test demonstrated that the high correlation coefficients associated to the NN approach are likely related to the well-known overfitting problem. The blind test also demonstrated that the TRPM approach, thanks to its general validity, often ensures a higher prediction capability than the empirical, data-driven, approaches. This high prediction capability and the general validity are the main advantages of theoretical rock physics models with respect to the empirical methods. The prediction capability of theoretical rock physics models has been demonstrated worldwide and it is discussed in several papers (e.g. Avseth *et al*. 2005). However, such prediction capability is high in case of sedimentary basins formed by alternating shales-sand sequences subjected to hydrostatic pressure and mechanical compaction regime. In more complex geologic scenarios (fractured rocks, non-clastic rocks, overpressured rocks and chemical compaction regimes), the theoretical rock-physics models become very complex and their prediction capability decreases. In these cases, empirical, data-driven, approaches could be the only chance to obtain a reliable rock-physics model applicable for reservoir characterization.

The previous considerations enable us to consider the rock-physics models derived by the SR and by the TRPM approaches in the following petrophysical-seismic inversion discussed in the companion paper "*Seismic reservoir characterization in offshore Nile Delta. Part II: Probabilistic petrophysical-seismic inversion*".

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