MULTI-MODE MULTI-OFFSET PHASE ANALYSIS OF SURFACE WAVE DATA (MMMOPA)

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Introduction. Seismic surface waves have started to be analysed in the late 1950's, with the purpose of retrieving useful information about the subsoil (Jones, 1958; Jones, 1962; Ballard, 1964). In the 1980's spectral methods for surface waves analysis were introduced and applied in the context of geotechnical studies (Nazarian and Stokoe II, 1984), but it is only in the late 1990's that the surface wave methods gained popularity, thanks to the development of multichannel techniques (Park *et al*., 1999; Xia *et al*., 1999).

Both active and passive techniques have been developed. Among the active, the Multi-Offset Phase Analysis (MOPA) (Strobbia and Foti, 2006) is a fairly recent technique for evaluating seismic surface wave dispersion and estimating the presence of lateral variations. Despite its robustness, MOPA has a limitation: it is based on the assumption of one predominant mode, usually the fundamental mode, in the wave propagation. If this condition is not satisfied, MOPA can still be used if the different modes are well separated: a preliminary processing step is required, which consists in isolating each mode, for example by filtering the data in f-k domain. Otherwise, when the different modes are close to each other and/or lateral variations are quite important, isolating modes prior to the analysis might be not applicable.

We present here a new approach, which can be considered as an extension of MOPA to the two-modes case, with possible extensions to more modes. The Multi-Mode MOPA (MMMOPA) is based on the concurrent analysis of both amplitude and phase, and is capable of extracting dispersion curves for at least two different interfering modes.

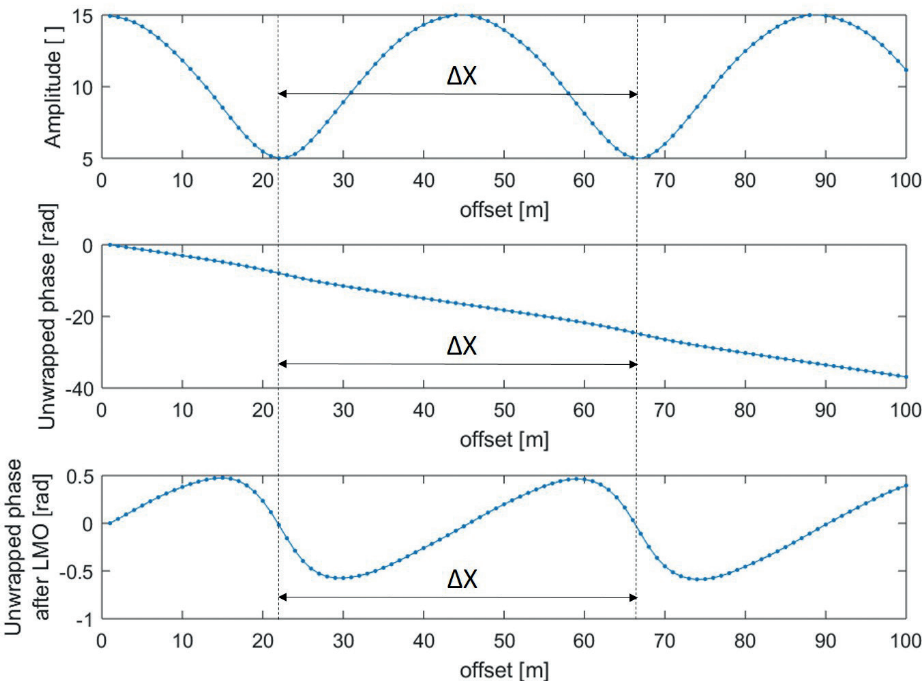

Fig. 1: From top to bottom: amplitude, unwrapped phase and unwrapped phase after LMO correction for the interference between two sinusoidal signals, a and b, with frequency f = 6 Hz, amplitudes $A_a = 10$ and $A_b = 5$ and apparent propagation velocities $c_a = 100$ m/s and $c_b = 160$ m/s. Velocity used for LMO is $v_{LMO} = 100$ m/s.

The method. In order to highlight the main concepts underlying the proposed approach, let us consider a simple theoretical case: the interference between two sinusoidal signals, a and b, with identical frequency and different amplitude, and yet propagating with different apparent velocities. Note that no specific reference is here made to surface waves, but only to the combination of two monochromatic waves. The amplitude presents a regular oscillating pattern with offset, due to positive and negative interference between the two waves (beats). Conversely, the phase presents regular smooth jumps in conjunction with amplitude minima, at the positions of phase opposition, better visible with a linear-move-out (LMO) correction applied (Fig. 1).

The example above clear show, to no surprise, that the combination of two different monochromatic signals having the same frequency produces observable patterns on seismograms. In particular:

1. Amplitude maxima and minima give information about the sum (constructive or in-phase interference) and the difference (destructive or out-of-phase interference) of the individual amplitudes of the two interfering waves. Conversely, we can infer the amplitudes of the single sinusoids, A_a and A_b, from the observed A_{max} and A_{min}:

$$A_a = \frac{\left(A_{max} + A_{min}\right)}{2} \tag{1}$$

$$A_b = \frac{\left(A_{max} - A_{min}\right)}{2} \tag{2}$$

2. The oscillating spatial period of amplitude and phase, let us call it ΔX, is linked to the difference of wavenumber (i.e. to the difference in velocity) between the two signals. In formula:

$$\Delta \mathrm{x} = \frac{2\pi}{(k_a - k_b)} \tag{3}$$

where k_a and k_b are the individual wavenumbers of the two sinusoids.

3. The local slope of the phase versus offset, k_{loc}, corresponding to the position of amplitude maxima, is equal to the weighted mean of the two individual slopes k_a and k_b, where the weights are given by the sinusoid amplitudes:

$$k_{loc} = \frac{\left(A_a k_a + A_b k_b\right)}{(A_a + A_b)} \tag{4}$$

Once we compute individual wave amplitudes, A_a and A_b, from equations 1 and 2, we can retrieve the phase slopes, or wavenumbers, from the solution of the system of equations 3 and 4. We have:

$$k_b = k_{loc} - \frac{2\pi}{\Delta X}\frac{A_a}{A_a + A_b} \tag{5}$$

$$k_a = k_{loc} + \frac{2\pi}{\Delta X}\frac{A_b}{A_a + A_b} \tag{6}$$

The concepts above, illustrated in the simple case of interference between two monochromatic waves propagating (in the same direction) with different velocities, can be easily extended to surface waves and, specifically, to the interference between two different modes. In fact, when using MOPA the surface wave analysis is performed independently for each frequency. Therefore, given a 2D seismic record with laterally homogeneous subsurface conditions (a vertical 1D model), it is possible to extract, for each frequency, the wavenumber information of two interfering modes, which consists in retrieving the multi-modal dispersion curves for that site. However, with surface waves and especially with real data, the presence of noise, lateral shear velocity variations, the interference of further (more than two) modes and the combination of geometrical and intrinsic attenuation could compromise the success of the method. Given these reasons above, we developed a robust and effective algorithm which can be applied to real data under a certain set of conditions. When those conditions are not met we can, in some cases, pre-process the data in order to make them still suitable for the Multi-mode MOPA. In the following sections we will discuss the application of the algorithm to both synthetic and real cases.

Results with synthetic data. A synthetic seismogram was generated using PUNCH (Kausel, 1989), a program based on a finite element solution in the direction of layering. For this reason, PUNCH can handle only 1D vertical velocity models, which is sufficient for the required demonstration. Note that in this work we do not consider lateral velocity variations, that will be the focus of a future piece of work.

In the simulation we applied a single shot using a point-load source at the surface. The receiver array is composed of 81 vertical geophones, spaced 1 m along a line with minimum and maximum offsets of 20 m and 100 m. We adopted a 1D shear wave velocity profile with a very fast velocity layer at shallow depth, between 3 and 5 m. Velocities above and below the concrete block are assumed to progressively increase with depth. The presence of a fast layer surrounded by low-velocity materials produces a strong velocity inversion, that in turn is known to generate higher modes with high energy (Boaga *et al.*, 2014).

From the analysis of the frequency normalized f-k spectrum (Fig. 2) we observe that in the range 15-30 Hz, most of Rayleigh wave energy is distributed between the fundamental and the first higher mode. For this reason, the Multi-Mode MOPA has been has been tested within this range. The analysis was performed in a fully automatic fashion using the algorithm mentioned in the previous section. The whole procedure is performed for each frequency independently.

First of all, the algorithm searches for a positive periodic function (the norm of a sinusoid) which approximates the observed data (Fig. 2). From this first step we obtain, for each frequency, the value of ΔX and the offset locations of amplitude maxima and minima.

Secondly, the algorithm extracts the individual amplitudes of the two modes (call them A_0 and A_1). In order to do so, amplitudes are first corrected for geometrical spreading and intrinsic attenuation. Then, amplitude minima and maxima are computed as the weighted, with respect to offset, average of all minima and maxima found in the previous step. The weight is advisable

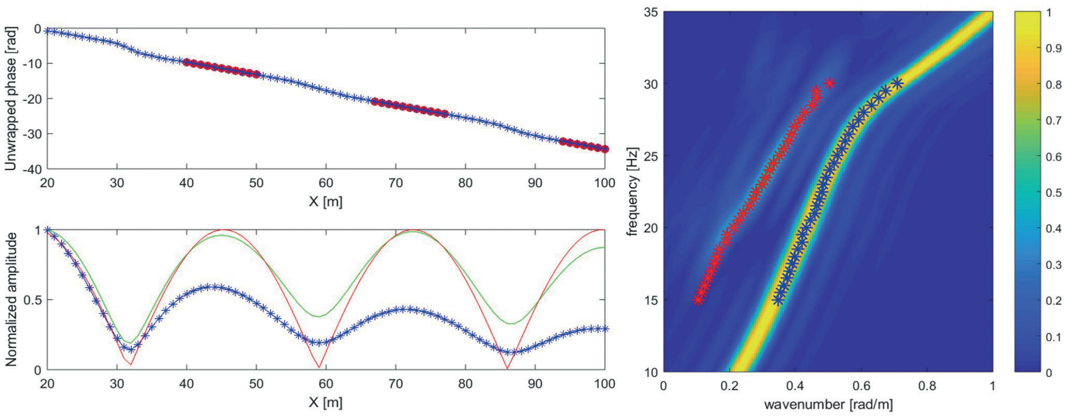

Fig. 2: Example with synthetic data. Left-upper panel: unwrapped phase at 19 Hz (blue stars) with values selected for each independent k_{loci} computation (red circles). Left-bottom panel: Normalized amplitude at 19 Hz (blue stars), normalized amplitude after correction for attenuation (green line), fitting sinusoid (red line). Right panel: f-k spectrum overlapped by computed fundamental (blue stars) and higher order (red stars) modes.

since data are affected by larger uncertainties at greater distance. By using equations 1 and 2 we finally derive A_0 and A_1 for each single frequency.

Thirdly, the local slope (or wavenumber) is retrieved. The developed algorithm takes the unwrapped (in offset) phase in a neighbourhood of each amplitude maximum i (Fig. 2), and computes a linear regression as in the classic MOPA approach (Strobbia and Foti, 2006). In formula:

$$\varnothing = G \bullet M \tag{7}$$

where G is the data kernel matrix, containing the offset information, Φ the phase vector and M the vector containing the unknown polynomial coefficients, $-k_{loci}$ and ϕ_{0i}. Using a least-squares approach we obtain M as:

$$M = G^{-g} \bullet \varnothing \tag{8}$$

being $G^{-g} = (G^T G)^{-1} \bullet G^T$ the pseudo-inverse of G. The value of the final local wavenumber k_{loc} for each single frequency is then computed as the weighted (with respect to offset, as done for amplitude) average of all k_{loci} values. This is possible since we are performing a 1D analysis (no lateral variations).

Finally, dispersion curves for the two co-existing modes were calculated using equations 5 and 6.

The application of the method to our synthetic dataset has given promising results. By overlapping the final dispersion curves on the normalized f-k spectrum we observe a perfect match between the retrieved dispersion curves and the peaks of maximal energy corresponding to the fundamental and first-order Rayleigh modes (Fig. 2).

Results with real data. Multi-mode MOPA has been applied to a 2D real dataset with laterally uniform conditions. The site is located in Mirandola, Italy, and it is one of the three acquisition sites of the InterPACIFIC project (Garofalo *et al.*, 2016). Our analysis was focused on the 1 meter spaced 48 channels active acquisition and, in particular, on the first shot location (energized 10 times). Traces were recorded with 4.5 Hz natural frequency vertical geophones, with a length of 2 s and a sampling interval of 0.25 ms.

From the observation of the f-k spectra we identified a possible frequency range for the analysis: between 10 Hz and 20 Hz the fundamental and the first higher modes are both present and quite energetic (Fig. 3). In this case, since we were using real data, amplitude distribution with offset was quite noisy: a light smoothing was necessary before fitting it with the periodic function. Uncertainties have also been estimated by propagating the error through the different steps of the analysis.

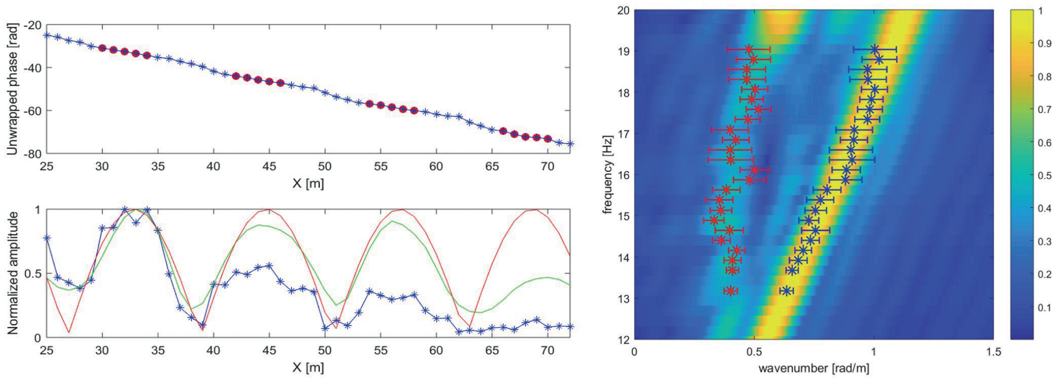

Fig. 3: Example with real data: Left-upper panel: unwrapped phase at 19 Hz (blue stars) with values selected for each independent k_{loci} computation (red circles). Left-bottom panel: Normalized amplitude at 19 Hz (blue stars), normalized smoothed amplitude after correction for attenuation (green line), fitting sinusoid (red line). Right panel: f-k spectrum overlapped by computed fundamental (blue stars) and higher order (red stars) dispersion curves. Error bars define uncertainty in the velocity estimation.

The results show a good match between 13 and 19 Hz (Fig. 3): at lower frequencies the two modes are not sufficiently separated, having very similar velocities, while at higher frequencies a third energy spot appears between them, making the analysis, based on two modes only, obviously inadequate. Uncertainties are also small and acceptable. This example shows how the Multi-Mode MOPA can correctly perform when the applicability conditions are met, which in this specific case verifies in a quite narrow frequency range. Noise is affecting the results accuracy, especially regarding higher frequencies, and this is correctly accounted for by the computed uncertainties.

Conclusions. A new approach, which makes it possible to apply the Multi-Offset Phase Analysis (MOPA) to two-modes situations, was here presented: we called it Multi-Mode Multi-Offset Phase Analysis (MMMOPA). The method is based on the analysis of both phase and amplitude spectra, which represents a main step forward with respect to the classic MOPA.

MMMOPA was tested on both synthetic and real datasets, with success. This work only aims at presenting the core of the method, with a set of strict applicability conditions: the analysis is focused on laterally homogeneous media, where two (and only two) modes are interfering. We also focused our attention on the single shot point. Future developments of MMMOPA will be oriented towards a generalization of the method, starting from a 2D approach. In order to do so data redundancy of multi-shot schemes should be exploited to compensate the sparse character of the method, where measures are done in conjunction with amplitude maxima only.

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